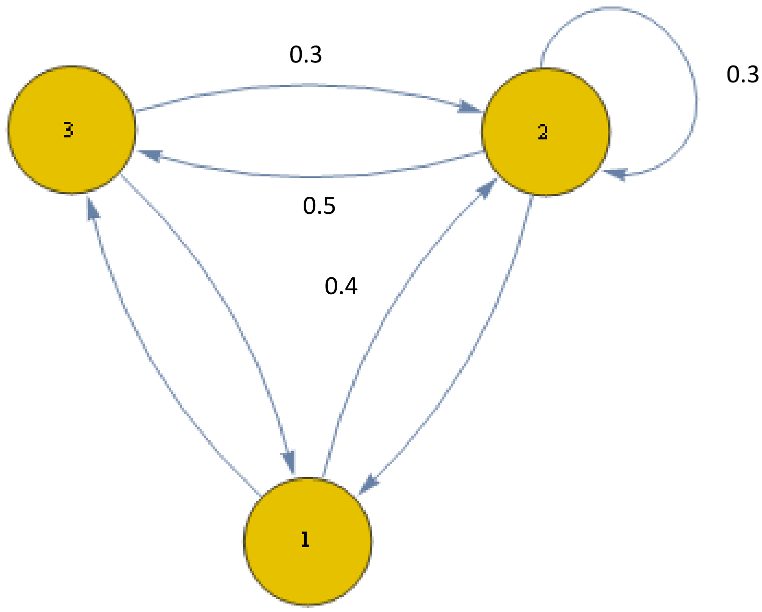
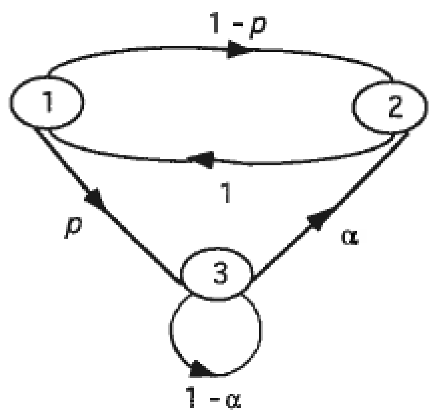
EECS 863
Homework 1



1. The state transition diagram for a homogeneous discrete time Markov chain is given above:

a) Find the missing conditional probability values.
b) Given the starting state is state 2 plot the probability of being in state 3 as a function of the n^{th} time step. Consider n = 1...10.
c) Repeat part b) changing the initial state to state 3.
d) Repeat part b) when the probability of being in state 1, 2, or 3 is 1/3 at n = 0.
e) Solve for the state probabilities as $n \rightarrow \infty$.
f) Plot 5 member functions of the discrete time Markov random chain defined by the state transition diagram given above. The y-axis should be the state and the x-axis the time index n. Consider n =1...10. (You may want to write a simple computer program to obtain these member functions.)

2. Consider the homogeneous discrete time Markov chain whose state transition diagram is given below:



a) Find **P,** the probability transition matrix.
b) Under what conditions (if any) will the chain be irreducible and aperiodic?
c) Solve for the equilibrium probability vector π.
d) What is the mean recurrence time for state 2?

3. In some networks the time axis is slotted with the packet length is equal to the slot time. In these systems the traffic can be modeled using discrete time Markov chains. The traffic source alternates between an active state and an idle state. In the active state the source generates a packet in each time slot while in the idle state no packets are generated. The probability of going from the active to idle states is P_{ai} while the probability of going from the idle to active state is P_{ia}. A Discrete Time Markov Chain is often used to model this packets arrival process. A single source with these characteristics called a binary source.

Assume the P_{ai}= 0.7 and P_{ia}= 0.4.

a) Consider a single binary source and draw a state transition diagram for the discrete time Markov Chain that defines the arrival process. Include all transition probabilities.
b) Find the transition probability matrix for the arrival process given in part a).
c) Find the steady-state probabilities for the arrival process give in part a).
d) Find the average number of arrivals in a time slot given in part a).
e) What is the probability of a burst of 5 consecutive packet transmissions?
f) What is the probability of a burst of 5 consecutive idle time slots?